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Smoke observations of the wake of a group of three cylinders at low Reynolds number

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A smoke visualization technique was used to study the laminar wake behind a group of three cylinders. The main characteristic of the interaction between the three cylinders was the appearance of strong sinuous oscillations some distance downstream in the wake, which led to the formation of a new single vortex street. The mechanism of the formation process of this vortex street and the part played in it by the sinuous oscillations is demonstrated by a series of photographs. The effects of variation of the spacing of the configuration have been examined in detail.

In an interaction between three fully developed vortex streets, some of the rows of vortices crossed, and there was an extremely complicated re-arrangement of vorticity in the wake.

Transition to turbulence in such wakes was also studied. The most interesting result observed was an asymmetric transition. Wakes were sometimes observed which were laminar on one side and turbulent on the other.

1. Introduction

While numerous investigations have been made of the flow past single obstacles having continuous or discontinuous surfaces, and of many diverse shapes, few studies have been made of the vortex pattern associated with complex configurations consisting of more than one obstacle. A natural extension of the study of alternating vortices is to groups of cylinders, of which the simplest case is that of a pair of cylinders separated by various distances. Two very different configurations of a pair of cylinders are the cases in which their axes lie in a plane parallel to the flow and in a plane perpendicular to the flow. In both cases the combination has two axes of symmetry at right angles. In the first case one of the cylinders is always in the wake of the other so we can say that they are subject to longitudinal mutual interaction. In the second case, both cylinders generate their own wakes, but if their lateral spacing is small enough we have lateral interaction of their wakes.

The simplest configuration with only one axis of symmetry is a group of three cylinders. Here longitudinal and lateral interactions appear simultaneously to a degree which depends on the geometry of the configuration. The main purpose of this study is the observation of such wakes, with particular interest in the formation and interaction of the vortex streets.

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2. Previous investigations

Taylor (1924) noted that when a four-pronged toasting fork was waved in the air, the singing noise produced was much louder when the prongs were in the plane of the motion than when they were perpendicular to the direction of the motion.

Biermann & Herrnstein (1933) tested various arrangements of streamline struts and cylinders at Reynolds numbers, between 25,000 and 140,000. They measured an 'interaction drag' defined as the difference between the drag of bodies in combination and the sum of the drags of the single bodies. In the case of circular cylinders for the side-by-side position with spacing less than two diameters the interference drag became negative. For the tandem position, however, the interference drag was always negative, and at spacings less than three diameters reached such a high negative value that the over-all drag of the cylinders was less than that of one isolated cylinder. The downstream cylinder experienced a forward thrust.

Spivack (1946), varied the spacing of two cylinders in a plane perpendicular to the flow, and observed the effect on vortex-shedding frequency with a hot-wire anemometer. Reynolds numbers were from 5000 to 93,000. When the cylinders were more than one diameter apart no interactions were detected and each cylinder acted as an independent body. For closer spacings, the frequency of vortex shedding of the outer rows of vortices decreased. That of the vortex rows behind the gap increased until a critical gap-spacing was reached at which a discontinuous fall in frequency occurred to a value lower than that of the outer rows. Spivack was not acquainted with Biermann & Herrnstein's paper, but Hoerner (1958) plotted their results together, as curves of Strouhal number and drag coefficient versus spacing, and showed that the abrupt change in frequency is closely related to the change in drag.

Hori (1959) investigated the more general case. A pair of cylinders with parallel axes was inclined at different angles of incidence to the free-stream velocity direction laying in plane perpendicular to them. The Reynolds numbers were from 300 to 12,000. The variation of shedding frequency, the pressure distributions around the cylinders and the mean velocity distribution in the wake were calculated.

Thomas & Kraus (1964) produced photographs of the wake behind two cylinders in tandem at Reynolds number 62. Contraction, expansion, cancellation and coalescence of the eddies occurred at different values of the cylinder spacing at the same speed.

Finally, a wide variety of papers has been published as a result of the increasing importance, from the engineering point of view, of vortex-induced vibrations in heat exchangers and coolers: Livesey & Dye (1962), Roberts (1966), Putnam (1959), Mabey (1965), etc. The authors of all these papers believed that the flow through in-line or staggered multi-tube banks depends on the Kármán vortex-street shedding frequency which applies to an isolated cylinder.

It is interesting to note that till the present there has been no attempt to investigate the simplest multi-tube configuration, namely, the group of three cylinders, in the laminar flow regime. The aim of this paper is to approach the problem from that direction in the hope that some of the observed mechanisms will be true in the turbulent regime as well.

3. Experimental arrangement

The present investigation was conducted in the Aerodynamics Laboratory of the Engineering Department, Cambridge University, in a vertical low-speed wind tunnel with upward flow. The working section of the tunnel is 12×6 in. (30×15 cm) in cross-section and 20 in. (50 cm) long. Hollow cylinders of diameter 0·043 in. (1·09 mm), spanning the shortest dimension of the working section were used to generate the vortex streets. They were fixed in holes of two spacers located at the wall and glass-window of the working section. The stiffness of the whole system as a cantilever prevented oscillations in flow regimes.

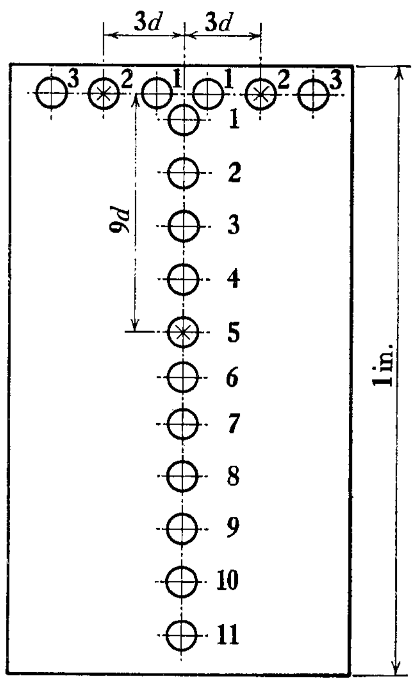

FIGURE 1. Nomenclature for the configuration of three cylinders.

The Reynolds number based on cylinder diameter and free-stream velocity was between 60 and 300. Precise numerical values of the Reynolds numbers will not be given in this paper because the phenomena observed depend on the level of turbulence in the tunnel, as was observed by Roshko (1954) for the case of a single cylinder. All the results presented here are qualitative in character.

The vortex street was made visible by the introduction of a 'smoke' filament into the free stream from an injector tube of diameter 0·075 in. (1·9 mm) located 8 in. (20 cm) upstream of the cylinder, near the end of the contraction. The visible material in what is here called the smoke filament was actually liquid kerosene in the form of a fog of fine droplets; it was produced by a Cambridge smoke generator of the Preston & Sweeting (1943) type as described by Maltby & Keating (1962). The wake configurations were photographed on ASA 400 film with a Nikon camera, with two electronic flash units each of about 100 Joules output as illumination. In the figures presented here the cylinders appear at the bottom of the photographs as in the wind-tunnel working section; they are partly indistinct because the depth of the focus of the photographs was small.

The configuration of the group of cylinders was varied in the experiments. The notation adopted to describe the configurations was as shown in figure 1, which represents one of the spacers in which cylinders were mounted (visible in some of the photographs). The pitch of the holes in the spacers was two diameters. The notation 9; 3, 3 represents the case when the upstream cylinder is in the fifth hole and the downstream pair in the holes numbered two; thus the axes of the latter were six diameters apart, and the upstream cylinder was nine diameters ahead of them. The same configuration reversed is denoted by 3, 3; 9. The angle between the axis of symmetry of the configuration and the direction of the free-stream velocity is here called the angle of incidence.

4. The formation of a new single vortex street in the wake

The main interest of this study was in the vortex systems generated in the wake of the group of three cylinders. As the result of their interaction, the wake significantly differed from that behind a single cylinder, well known as the Kármán vortex street.

The three main features observed, which will be described in detail further below, were: (i) the appearance of well-defined vortices only at the outside of the wake; (ii) very rapid decay of these vortices although the wake was fairly straight; (iii) the appearance of sinuous oscillations some distance downstream in the wake which lead to the formation of a new vortex street.

Investigations were first carried out on the configuration 5; $1\frac{1}{4}$, $1\frac{1}{4}$ and the flow observed was as shown in figure 2, plate 1. (The Reynolds number was approximately 100.) As the result of mutual interaction between three cylinders, no Kármán vortex street was observed behind the upstream cylinder, and only the outer rows of vortices were seen behind the downstream pair of cylinders. The loss of the inner rows caused the spacing ratio of the vortices to be 0·75. The alternate shedding of these vortices could be explained by the sinuous motion of the flow approaching the downstream cylinder. That the upstream cylinder did not form vortices in the five diameters before the downstream cylinders is surprising. What Schaefer & Eskinazi (1959) had found was a formation region of less than three diameters at these Reynolds numbers behind a single cylinder.

The smoke filament passing the upstream cylinder continued downstream in a sinuous wake which can be thought of as two shear layers. As these shear layers passed between the downstream pair of cylinders new shear layers of opposite sign and nearly equal strength were formed very close to them. The central part of the wake was therefore a region of cancellation and decay of the pairs of shear layers of opposite signs through the influence of viscosity.

The process of the formation of the vortices in the outer vortex rows is mainly a matter of the rolling up process of the two outer shear layers, as is evident from the shape of the visible vortex cores. The decay process of rolled-up vortices and shear layers in the central part of the wake is seen to be straight in the initial region. However, at a distance downstream at which the major axes of the elliptically diffused vortices have become nearly parallel to the axis of the wake, an oscillatory instability of increasing amplitude can be seen. As a consequence of the lateral displacements of the vortices due to this oscillation, their longitudinal velocities have changed. Those nearer to the axis of the wake are retarded and those further away accelerated relative to the original velocities. This has the effect of bringing pairs of vortices abreast of one another in each of the outer rows. The mutual influence of two point vortices in potential flow is a rotation about their 'centre of gravity'. A similar rotation results from the interaction of two vortex 'clouds' here. This is followed by the formation of a new single vortex street. The details of the process can best be followed on the right-hand side in figure 2. It is interesting to note that the 'Phoenix-like' behaviour of the Kármán vortex street, which was observed by Taneda (1957), may be explained in the same way.

Since it is known that the flow in the formation region of the ordinary vortex street can be three-dimensional (Nayler & Frazer, 1917), it may be asked whether the same is true for this formation process. In some of the many photographs taken, for example figure 3, plate 1, three-dimensionality of the flow is evident, in that there are signs of a motion in planes perpendicular to the free-stream direction. This will be seen if very thin smoke lines are followed in figure 3 in their swirling. The newly formed vortices seem to be somewhat distorted by the swirling motion. In order to investigate whether the new vortex street could be suppressed altogether, a swirling motion was introduced into the smoke filament at its source; the effect on the wake is shown in figure 4, plate 1. It appears that the initial swirling flow in the free stream has prevented the formation of the new vortex street completely. A similar mechanism of suppression of vortex shedding was observed by Humphreys (1960) in the case of a single cylinder at Reynolds number 10^5. Naumann, Morsbach & Kramer (1966) also showed that at high subsonic speeds, when shock waves were present, the Kármán vortices were cancelled or their development was prevented by artificially introduced three-dimensional disturbance.

Finally, the configuration was reversed and investigated at zero incidence as the case $1\frac{1}{4}$, $1\frac{1}{4}$; 7. In figure 5, plate 2, the process of formation of a single vortex street is seen again. It is obvious that the presence of large black fragments in the newly-formed vortex cores was due to the smoke flow not filling the whole of the wake. Also the rolling-up of the outer vortices cannot be clearly seen because the smoke did not flow outside the upstream pair of cylinders.

5. Variation of the spacing of the configuration

The next thing to be changed was the spacing of the configuration. The photographs shown on plate 3 illustrate wakes typical of those observed. Before these pictures are described some general remarks have to be made.

Increasing the distance of the upstream cylinder from the downstream pair of cylinders had a twofold effect. First, for distances greater than some value which depended on the Reynolds number, a Kármán vortex street could be seen behind the upstream cylinder. It may be said that there was no longer a mutual interaction affecting all three cylinders, but that there was an interaction between the Kármán vortex street of the upstream cylinder and the flow about the two downstream cylinders. When the spacing was decreased below that limit, an increasing oscillation of the wake but no vortex street was observed between the upstream cylinder and the two downstream cylinders. Finally, for very small spacings, the group of three cylinders behaved more like a single body.

Secondly, when the downstream cylinders are in the wake of the upstream one, they are effectively in a stream with velocity reduced by an amount which depends on the spacings.

The effect of decreasing the spacing of the downstream pair from 2·5 to 2 diameters caused a considerable change in the wake geometry. The flow with a configuration 11; 1, 1 is shown in figure 6, plate 2, and several interesting features can be seen. The length of the zone in which the initial rolling-up of the shear layers occurs is much shorter. The sinuous oscillation of the wake begins almost immediately behind the cylinders. The newly formed vortex street downstream, shows a remarkable expansion: the vortex rows diverge at a considerable angle.

There is a possible connexion between the expansion of the vortex street here and the observations by Taneda (1965) and Koopmann (1967) of the wake behind a single vibrating cylinder. They found that vibration of the cylinder at the shedding frequency caused the vortex street to expand. In the case of the group of three cylinders, the cylinder upstream produces sinuous oscillations of the stream approaching the downstream pair. Thus instead of a vibrating cylinder in a steady free stream we have an oscillating free stream approaching the downstream pair of stationary cylinders.

When the configuration was changed in such a manner that the spacing between the downstream cylinders was increased, the wake geometry was drastically changed again. The flow with configuration 17; 1, 3 (placed symmetrically) is shown in figure 7, plate 3, for Reynolds number of the order of 80. The outer vortices are formed again but sinuous oscillations are not seen along the whole visible length of the wake. At a higher Reynolds number (of the order 120) the single vortex street is seen once again in figure 8, plate 3.

Generally speaking, the spacing between the downstream pair of cylinders has a major effect on the appearance of sinuous oscillations in the wake. As the spacing decreased from the value at which a significant interaction first appeared (4 diameters) the length of the non-sinuous part of the wake also decreased.

When the two cylinders finally touched one another the oscillations in the wake began immediately downstream, as is the case behind a single cylinder to which this configuration is qualitatively analogous. The length of the non-sinuous part of the wake depends at the same time on Reynolds number, increase of which shortens this length.

6. Variation of the incidence of the configuration

Varying the incidence produced a large number of different interactions. Fortunately, most of these interactions could be classified in three main groups: (i) those at small† angles of incidence of the axis of symmetry of the configuration (region (i) in figure 9); (ii) those in a range of angles of incidence

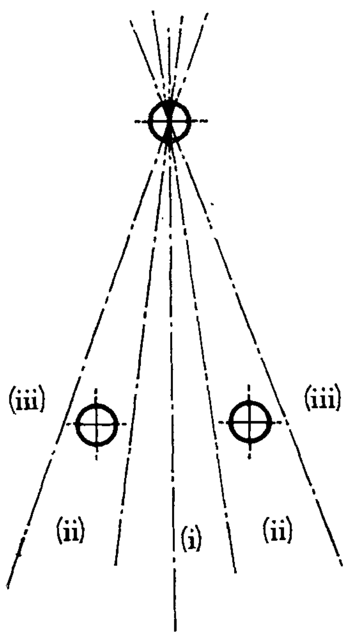

FIGURE 9. Nomenclature for the angle of incidence.

adjacent to region (i) (region (ii) in figure 9); (iii) those at large angles† of incidence.

The configuration 5; $1\frac{1}{4}$, $1\frac{1}{4}$ which had been investigated in §4, was used again at small angles of incidence. All three features previously observed, i.e. formation and decay of vortices originating by rolling-up of shear layers, the sinuous instability and the formation of a single vortex street are seen again in figure 10, plate 4 (non-reverting configuration). Unfortunately in this photograph only one vortex row of the new street is displayed. The width of the

† The descriptions 'small' and 'large' are only relative, because the absolute values of the angles of incidence depend on the ratio of the distance between the downstream pair to their distance from the single cylinder. As this ratio is increased the absolute values of the angles bordering regions (i) and (ii) decrease. In the experiments presented here this ratio was always bigger than 1·5.

smoke filament was not sufficient to reveal the whole wake. Consequently in figure 10 the wake is visualized only behind one of the outer cylinders.

It was possible to spread the smoke into almost the entire wake by a slight change in the configuration to 3; $1\frac{1}{4}$, $1\frac{1}{4}$ (figure 11, plate 4). With the upstream cylinder so displaced, the breadth of the smoke filament at the portion of the downstream cylinder was greater, and attached vortices were observed behind the upstream cylinder ($Re \approx 60$). The angle of incidence was such that two cylinders were exactly in line, on the right-hand side. It can be seen that no vortices are formed on that side in the initial part of the wake. It follows that a second row of vortices is not necessary to allow the rolling-up process in the shear layer on the left. A complete vortex street is, nevertheless, formed downstream as before. This case represented the limiting case between first and second group.

The angle of incidence was now chosen such that the downstream cylinder was slightly out of the wake of one of the cylinders of the upstream pair. An unexpected result seen in figure 12, plate 4, was observed. The vortices produced by the initial rolling-up of shear layers were shed with different frequencies and different strength on the two sides. But again the frequencies of the two rows in the single vortex street formed downstream were equal.

The second representative of the group (ii) was the case when a developed Kármán vortex street behind the upstream cylinder hit the downstream cylinder. The flow with inclined configuration 19; 3, 3 is shown in figure 13, plate 5. An interaction is seen of a Kármán vortex street which has developed upstream with an eccentrically situated downstream cylinder; the wake of the third cylinder has not been made visible. The core of a vorticity on the left-hand side—visible as smoke cloud—is cut in two as it passes the downstream cylinder. The left-hand part remains very close to the cylinder surface and participates in the formation of the new left-hand vortex. The whole of the right part moves further away causing right-hand vortex to be formed behind the downstream cylinder. But transformations occurring in the wake change this state very quickly. The right-hand vorticity cloud of the upstream cylinder hardly touches the downstream cylinder as it passes by, but at a distance of one longitudinal spacing downstream it becomes pear-shaped and reinforces the vortex of the same sign. This process is nearly completed after the fourth pair of vortices where both rows seem to have the same strength. The remainder combines with the right-hand part of the left-hand vorticity cloud of the upstream cylinder to form another vortex street with rapidly decreasing spacing ratio. When they are approximately aligned in one row, an interaction begins between that vortex street and the right-hand row of the left-hand vortex street, leading to their mutual cancellation.

When the incidence is slightly changed and the downstream cylinder approaches the wake axis of the upstream one, only one vortex street is formed behind them as seen in figure 14, plate 5; the left-hand vortex street was behind the third cylinder at the side. The previously observed 'secondary' vortex street now degenerates into 'white tongues', which disappear completely when the cylinders are exactly one behind the other. In figure 14 the wake of the cylinder at the side had been made visible by introducing another smoke filament; the interaction between the vortex streets is displayed also where they approach very close during the widening process. Only one row of well-defined vortices remains, similarly to that observed in figure 13. The interaction of vortex streets will be dealt with in the next section.

Finally, case (iii) only nominally concerns a configuration of three cylinders; the wakes forming in this case are always only combinations of a Kármán vortex street and the wake of a pair of cylinders.

7. Interaction of the Kármán vortex streets

When the lateral spacing between cylinders was increased further, Kármán vortex streets were generated behind all three at zero incidence. A complex interaction took place downstream between the three vortex streets, as shown in figure 15, plate 6, for the configuration 19; 5, 5. The smoke picture of the wake was broken down in various ways in order to analyse what actually happens to the Kármán vortex streets. The wake behind the upstream cylinder only was made visible in figure 16, plate 6, by moving the other smoke filaments further aside. This middle vortex street begins to interact with the others immediately after passing between the downstream pair of cylinders, in such a manner that its two rows of vortices approach one another until all the vortices lie in a single row on the wake axis. The spacing is then zero and the shape of the smoke cloud is markedly changed. The further motion is characterized by separation of these two vortex rows again, but first they cross one another, i.e. the vortices of the left-hand row of the middle vortex street move over to the right-hand side and vice versa. Formally, the spacing ratio may be said to have become negative; physically, the flow downstream of the middle cylinder corresponds to that of a jet in this respect.

The same technique was used to analyse the behaviour of the outer vortex streets in the interaction. In figure 17, plate 6, a single smoke filament hit the right-hand cylinder and made the wake downstream of it visible. While the right-hand row of vortices in this wake remains well defined along the whole length of the picture, the left-hand row goes through a series of changing shapes. At first the left-hand vortices are elongated. As they cross the centre of the combined wake of the three cylinders they are retarded; for instance, at the same cross-section as the 20th right-hand smoke cloud we find the remainder of the 25th left-hand cloud. As can be seen from figure 17 the smoke material of the clouds initially at the left of this wake finds its way, with the smoke from the central part, to the outer regions of the combined wake on both sides.

A slight change in the angle of incidence of this configuration causes the interaction to be limited at first to the two vortex streets which are closest together, as shown in figure 18, plate 7. As seen at the top, further downstream the interaction spreads to the opposite vortex street also. At higher angles of incidence all three cylinders have different shedding frequencies, and this causes severe disturbances in the parts of the wake where the vortex streets are closest together. The disturbances are of a type very similar to that shown in figure 23, plate 8.

The same configuration reversed at zero incidence with the pair of cylinders upstream, 5, 5; 19, is shown in figure 19, plate 7. It is surprising to see that the middle vortex street remained parallel along the whole length of the picture. This is also shown in figures 21 and 22, plate 8, for the configuration 3, 3; 17 and 3, 3; 5 respectively. A change in the spacing ratio of the outer vortex street can be seen on the right-hand side in figure 20, plate 7, in the same manner as was observed for the middle vortex street in figure 15.

Another difference between the initial and reversed configuration will be emphasized. In the case of the 19; 5, 5 configuration the shedding frequency of the downstream pair of cylinders was always exactly in phase when the arrangement was approximately symmetrical. On the other hand, for the reversed configuration 5, 5; 19 in almost symmetrical arrangement the shedding was often neither in phase nor with the same frequency. Figure 23, plate 8, shows an asymmetric flow of this kind. Severe disturbances of the wake in the form of considerable displacement of the middle vortex street happen whenever the neighbouring vortex streets fall in phase with one another.

8. Transition to turbulence

The last stage of the observations was concerned with the transition of the wake to turbulence. Three different mechanisms were observed at different spacings of the cylinders; (i) symmetric transverse oscillations of the vortex rows; (ii) oscillations of individual vortices along an oval path; (iii) asymmetric interaction in the wake.

The configuration 5; $1\frac{1}{4}$, $1\frac{1}{4}$, whose laminar flow behaviour was described in detail in §4, was used again. It was found that when the velocity was increased ($Re \approx 150$) oscillatory motions appeared in the wake. The observed motions of the vortex rows were oscillations which were seen to occur over the whole length of the wake visible in the tunnel, in the form of long waves moving symmetrically towards and away from the centre plane in the manner illustrated in figures 24 and 25, plate 9. Once the amplitude of the oscillations became nearly equal to the distance between the vortex rows transition to turbulence always followed. When the wind-tunnel speed and the Reynolds number were raised further the transition moved upstream towards the cylinders, and finally brought the transition region immediately behind the cylinders ($Re \approx 200$). No formation of a new vortex street was observed in the turbulent region of the wake downstream. It is interesting to note that this mechanism was the same as observed behind the single cylinder by the same technique (Zdravkovich 1967).

As an example of the second kind of transition, the wake of the 11; 1, 1 configuration is shown in figure 26, plate 10. The locus of the positions of a vortex as seen with the stroboscope was an oval, whose size increased with distance downstream in the wake. When two neighbouring oval-paths touched one another the vortex flow was so distorted that it became turbulent. The interpretation of this observation was that the vortices were subject to longitudinal and lateral oscillations simultaneously.

The third kind of transition to turbulence observed was the result of asymmetry of the configuration of cylinders. The configuration 5; $1\frac{1}{4}$, $1\frac{1}{4}$ was inclined in such a way that the upstream cylinder and one of the downstream cylinders were in tandem with one another, with the third to one side. The asymmetric transition to turbulence then observed ($Re \approx 140$) is shown in figure 27, plate 9. The left-hand side of the wake remained laminar longer than the right-hand side. This asymmetry was possibly due to the less relative speeds of lateral spreading of turbulence in the region of the wake behind the tandem cylinders. A similar asymmetric transition to turbulence was observed when all three cylinders produced vortex streets. The configuration 19; 5, 5 was slightly inclined so that the left-hand and middle vortex streets were nearer to one another. As seen in figure 28, plate 10, this caused transition to turbulence earlier than on the other side of the wake.

9. Similarity of the phenomenon

Describing the configuration by quoting the ratio of longitudinal spacing to lateral spacing has deliberately been avoided in this account. The reason for this was that different wake patterns were observed at equal spacing ratios thus defined. For instance, the configurations 4; 1, 1, 12; 3, 3 and 20; 5, 5 produced completely different wake interactions with the same diameter of the cylinders. In fact, the ratios of the longitudinal or lateral spacing to the diameter of the cylinders were different in all three cases. Therefore the similarity between wakes should be observed only if the spacings and diameters of three cylinders were similar. This was confirmed by experiments with cylinders of various diameters. Beside the group of three cylinders of 0·049 in. diameter mentioned previously, two more groups with diameters 0·025 in. (0·63 mm) and 0·070 in. (1·78 mm) were used. The observed wakes behind them were similar to those previously shown when the Reynolds number and configuration were the same.

10. Conclusions

An enormous variety of wake patterns and features has been observed behind the group of three cylinders. It may be worthwhile to underline some of the more commonly observed features.

(i) The vortex systems behind three cylinders always decayed very quickly. At the same time sinuous oscillations of growing amplitude appeared in the wake. This was followed by the formation of an entirely new single vortex street. The formation process of a new vortex street could be explained by two coupled mechanisms. One is the instability of shear layers which leads to the sinuous oscillations and the other is the rolling-up process induced by the distributed vorticity in the wake. During the process of formation of the new vortex street the longitudinal spacing of vortices in the wake changed, and this meant that the frequency was altered. Thus the Strouhal number varied along the wake and a single Strouhal number did not define the periodicity in the wake.

This may perhaps explain the considerable scatter of data obtained in the case of banks of large numbers of tubes in turbulent flow. Most of the curves which have been given by various authors (listed at the end of §2) of the Strouhal number versus Reynolds number plots in the same speed range and with similar multi-tube arrangements differ considerably because of the different positions at which the frequency was measured. On the other side frequency measurements by means of strain gauges attached to the tubes have given much higher Strouhal number (Chen 1967).

(ii) When the lateral spacings between the three cylinders were sufficient three initially separate Kármán vortex streets were generated. The interaction which occurred downstream in the wakes was manifested at first in unexpected changes of the spacing ratios of the vortex streets as they continued downstream, leading to crossing of the vortex rows.

(iii) The chaotic smoke motion which we identify as turbulence began in the wake above a certain Reynolds number, at a distance from the cylinders at which symmetric transverse oscillations brought two rows of vortices together. As the Reynolds number was increased the wavelength of the symmetric oscillations descreased and the transition region moved upstream nearer to the body. This process had been also observed to occur downstream of a single cylinder. A somewhat similar transition was observed in the new single vortex street which was formed well downstream of the group of cylinders. The vortices were subject to lateral and longitudinal oscillations at the same time; thus the newly-formed vortex street was less stable and transition appeared earlier than in a primary vortex street. The oscillations were more pronounced when the lateral spacings of the configuration were small.

The interaction of shear layers which had been developed in different ways gave asymmetric transition to turbulence in the combined wake. This could be possible only if the lateral spreading of turbulence was negligible in comparison with the longitudinal one.

The experiments presented here were intentionally limited to qualitative observations owing to the difficult nature of the subject. Therefore all the conclusions are also qualitative.

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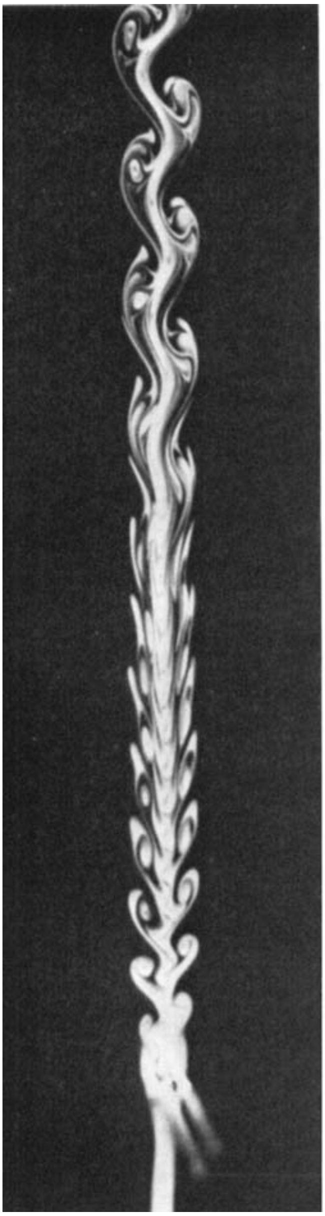

FIGURE 2

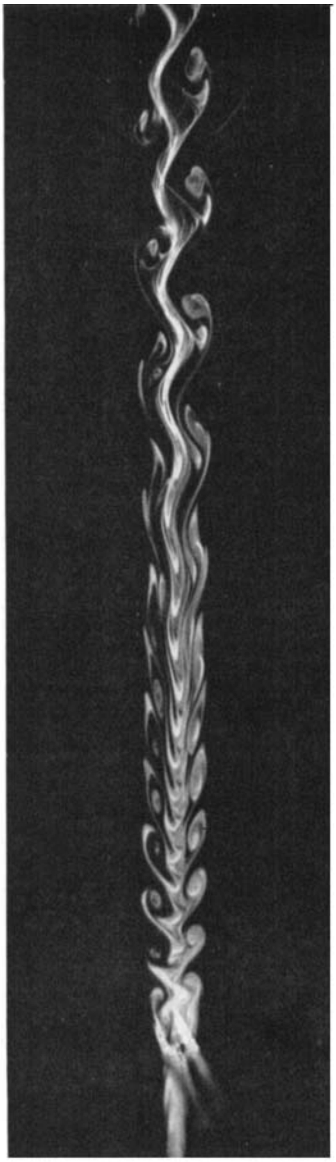

FIGURE 3

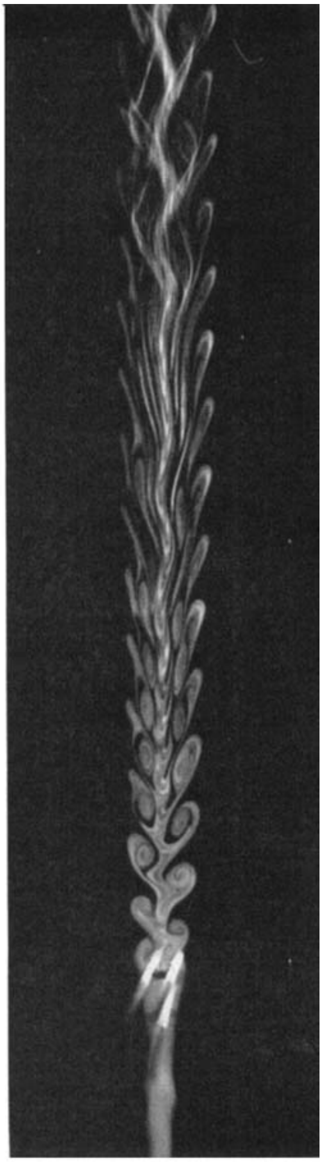

FIGURE 4

FIGURES 2–4. Configuration 5; $1\frac{1}{4}$, $1\frac{1}{4}$ ($Re \approx 100$).

FIGURE 2. Formation process of a new single vortex street in a wake.
FIGURE 3. Effect of longitudinal swirling flow on the wake.
FIGURE 4. Suppression of the formation of a new single vortex street by longitudinal swirling.

FIGURE 5

FIGURE 6

FIGURE 5. The wake behind a reversed configuration ($1\frac{1}{4}$, $1\frac{1}{4}$; 7).
FIGURE 6. The effect of a decrease of lateral spacing (11; 1, 1).

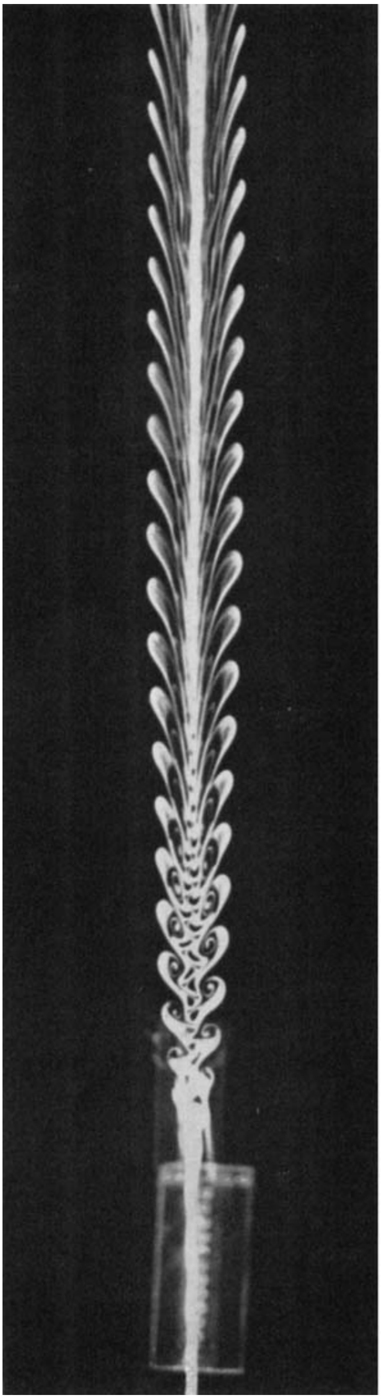

FIGURE 7

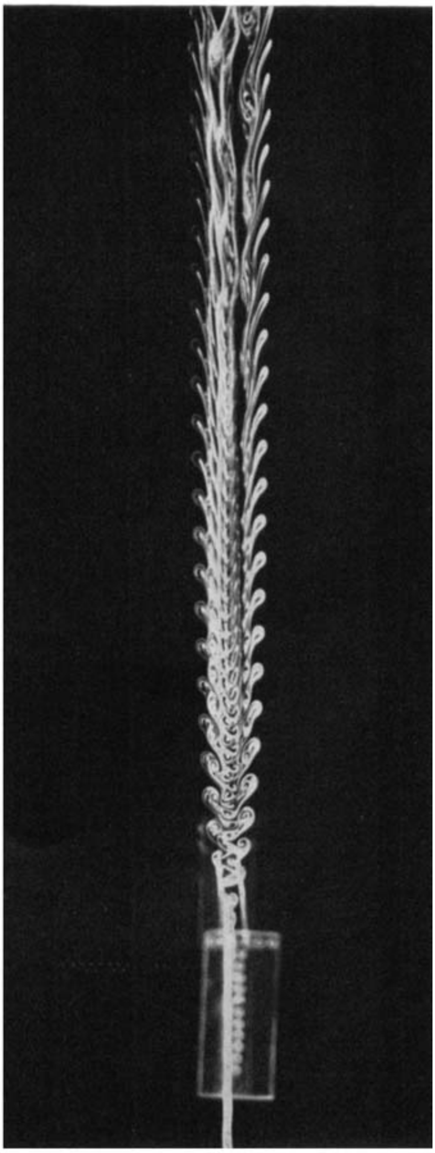

FIGURE 8

FIGURE 7. Effect of an increase of spacing (17; 1, 3) ($Re \approx 80$).
FIGURE 8. Effect of an increase of speed (17; 1, 3) ($Re \approx 120$).

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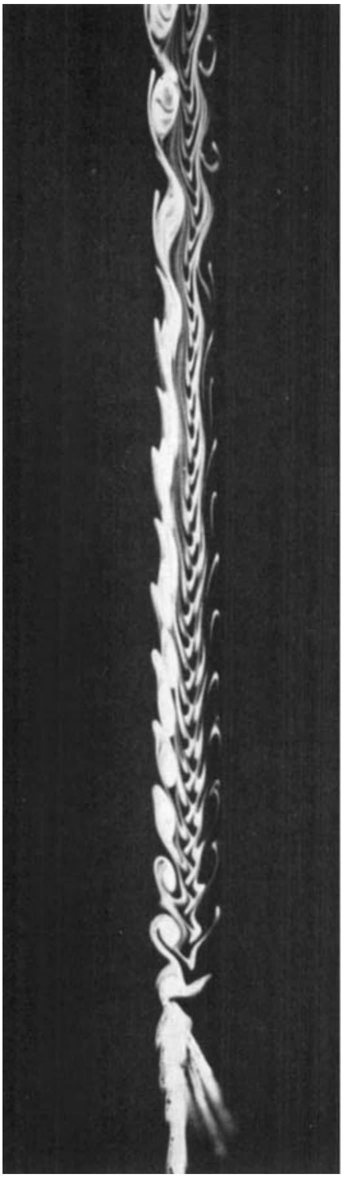

FIGURE 10

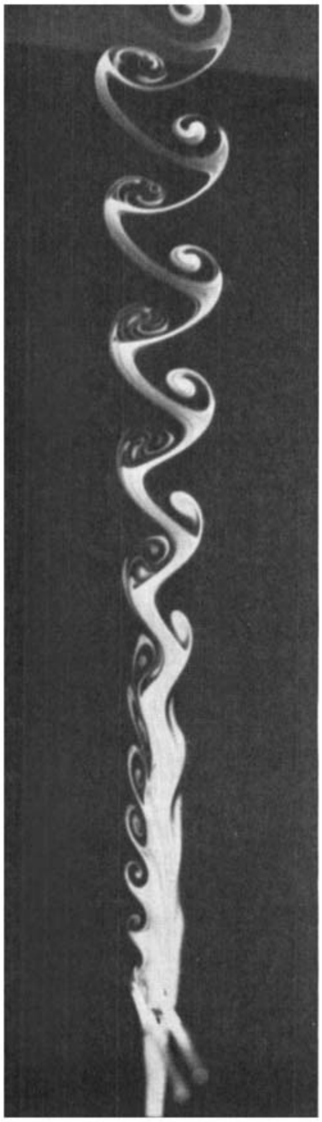

FIGURE 11

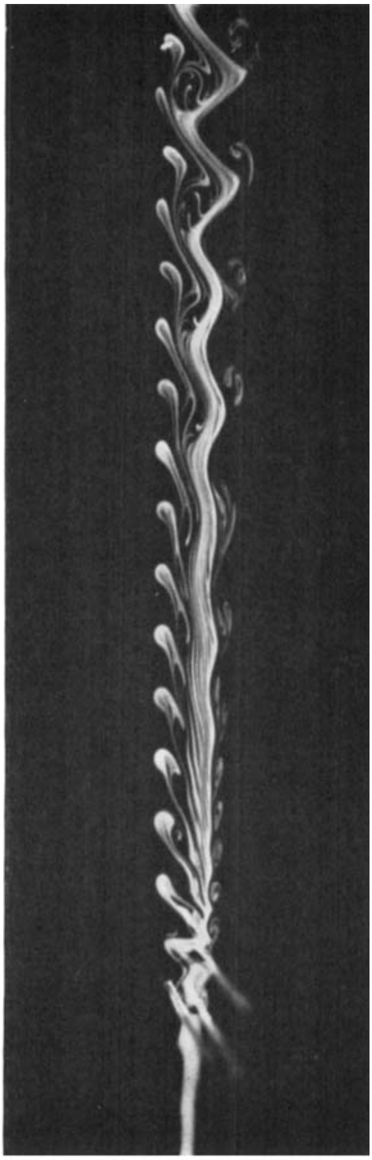

FIGURE 12

FIGURE 10. Effect of a small angle of incidence ($5; 1\frac{1}{4}, 1\frac{1}{4}$) ($Re \approx 100$).
FIGURE 11. Effect of a small angle of incidence ($3; 1\frac{1}{4}, 1\frac{1}{4}$) ($Re \approx 60$).
FIGURE 12. Effect of a bigger angle of incidence ($1\frac{1}{4}, 1\frac{1}{4}; 7$) ($Re \approx 100$).

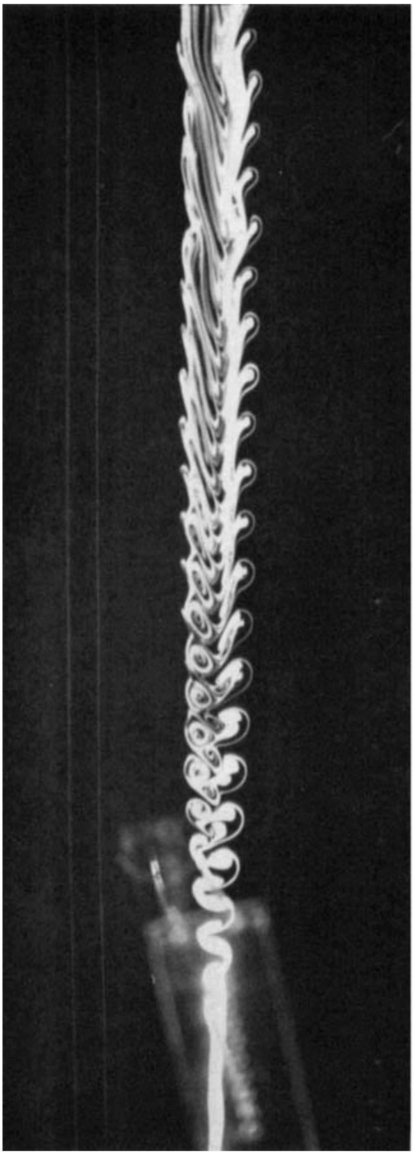

FIGURE 13

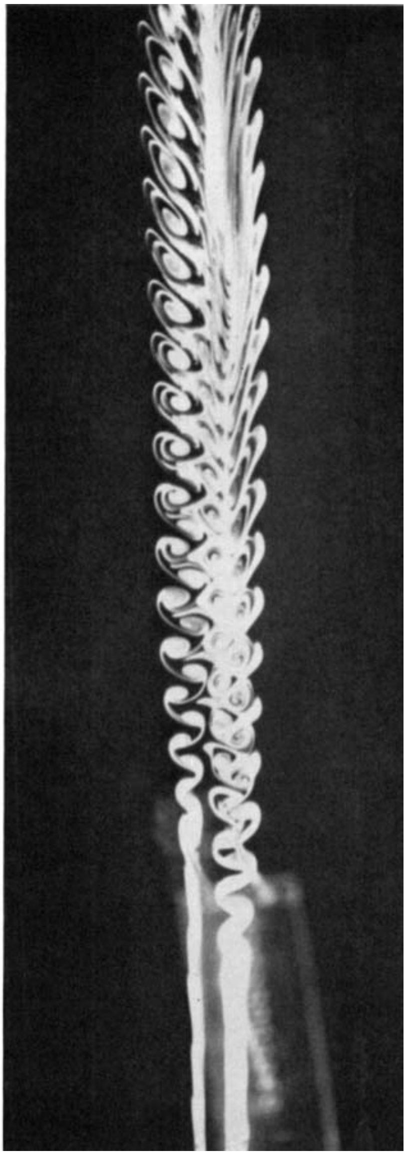

FIGURE 14

FIGURE 13. Effect of fully developed vortex street on the formation of another vortex street behind the downstream cylinder.
FIGURE 14. The same at a smaller angle of incidence.

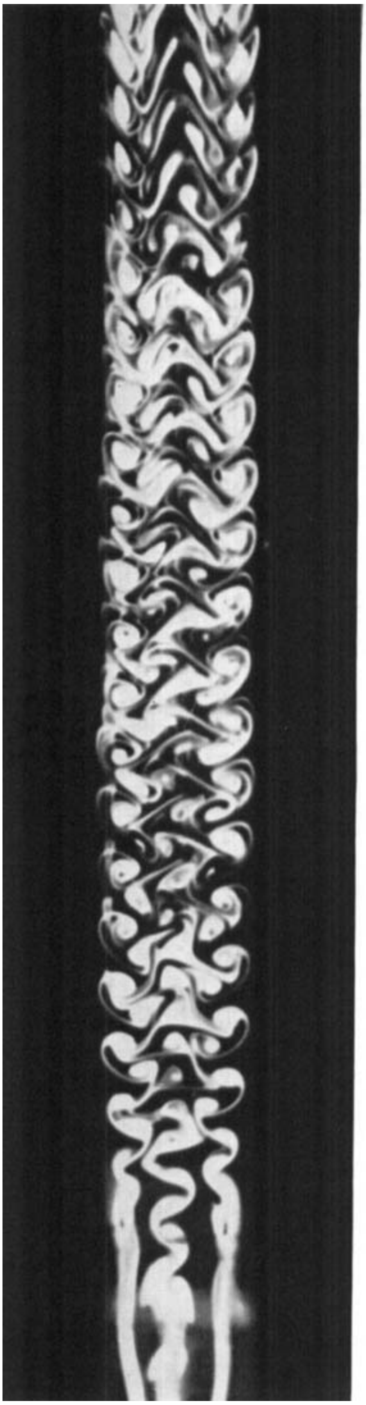

FIGURE 15

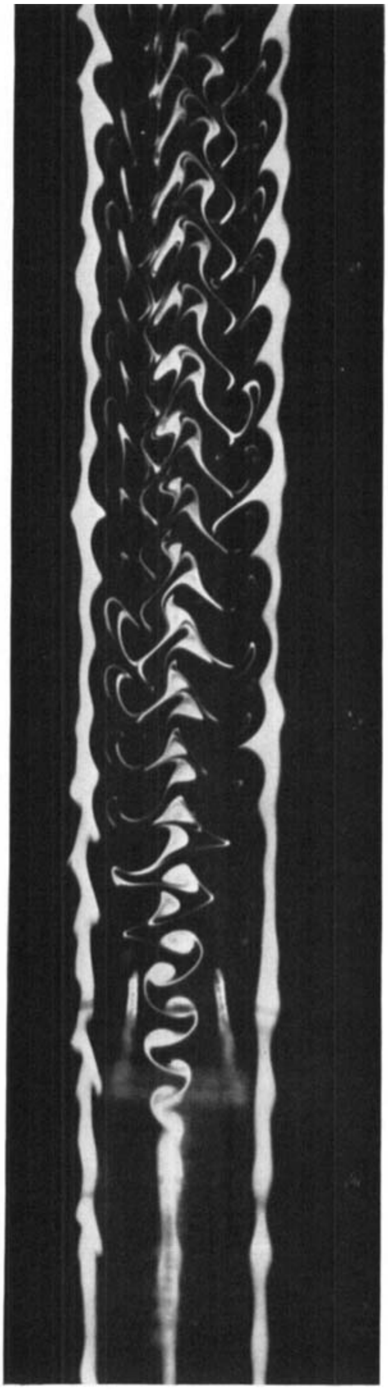

FIGURE 16

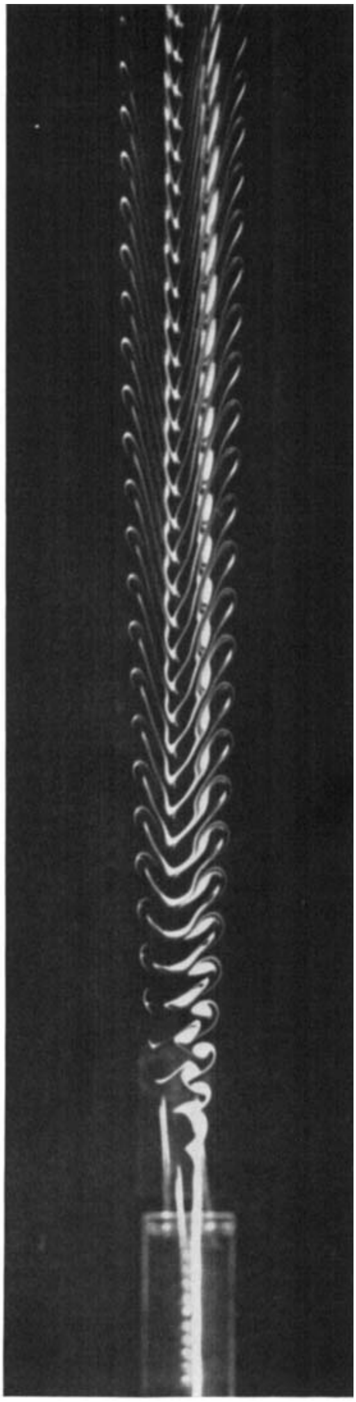

FIGURE 17

FIGURE 15. The interaction of three Kármán vortex streets (19; 5, 5).
FIGURE 16. Detail of the middle vortex street interaction.
FIGURE 17. Detail of the right-hand vortex street interaction.

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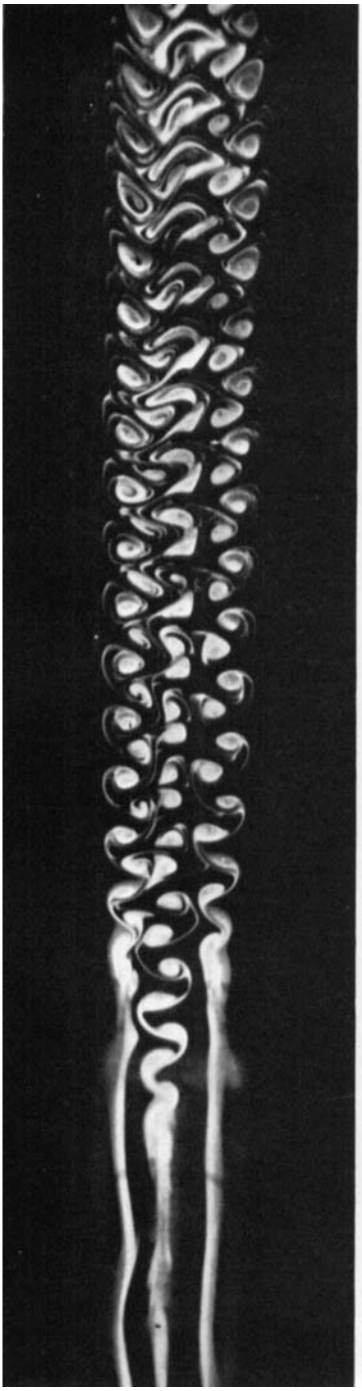

FIGURE 18

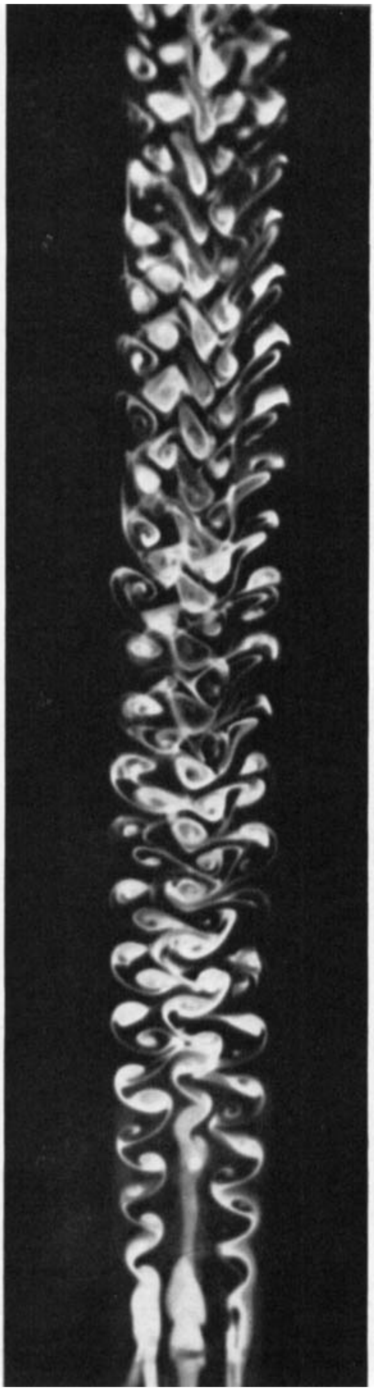

FIGURE 19

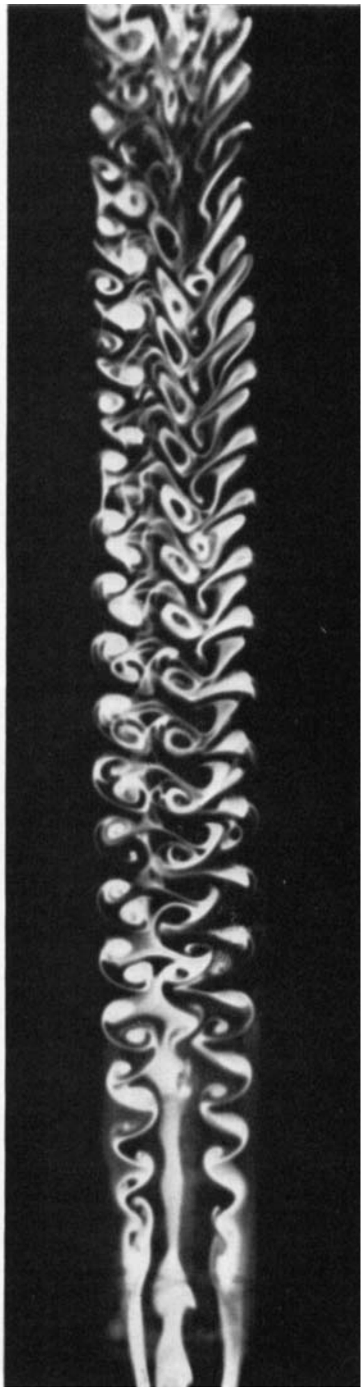

FIGURE 20

FIGURE 18. Effect of angle of incidence (19; 5, 5). Left-hand vortex streets are closer together.
FIGURE 19. The interaction for the reversed configuration 5, 5; 19.
FIGURE 20. The same with details of downstream development of the right-hand vortex street.

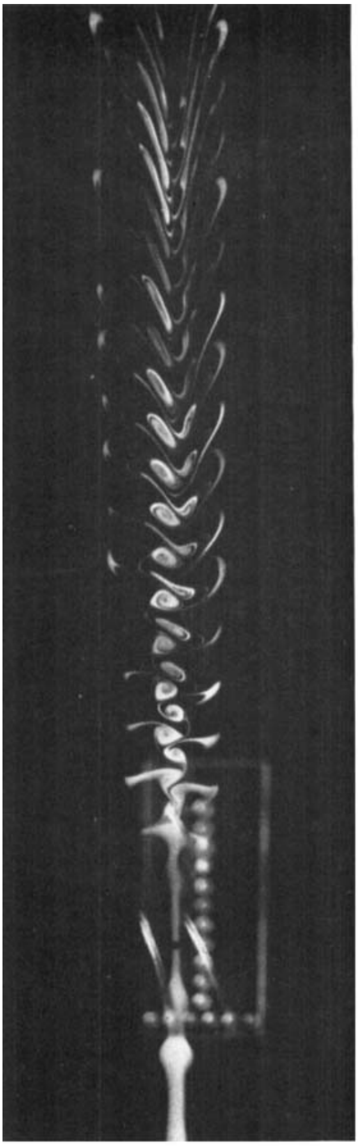

FIGURE 21

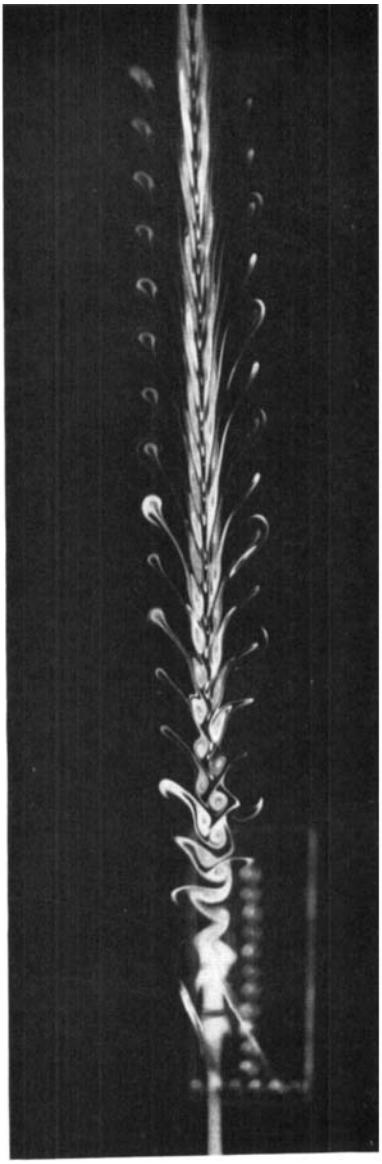

FIGURE 22

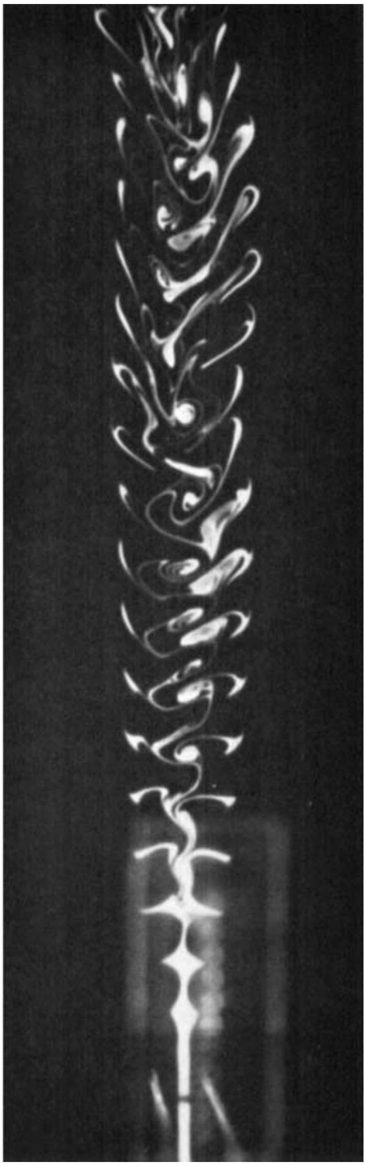

FIGURE 23

FIGURE 21. Detail of the middle vortex street interaction (3, 3; 17).
FIGURE 22. Detail of the middle vortex street interaction (3, 3; 5).
FIGURE 23. Middle vortex street with out-of-phase shedding by the upstream pair of cylinders (5, 5; 19).

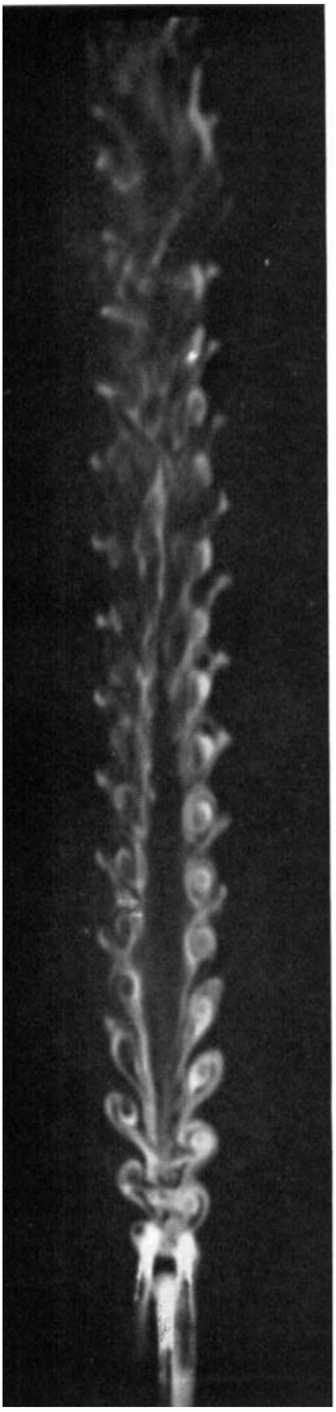

FIGURE 24

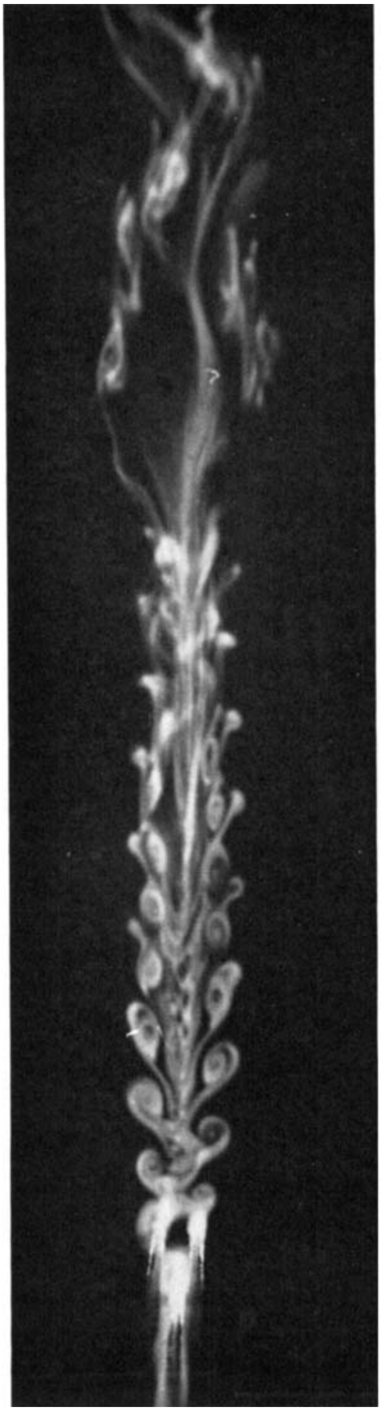

FIGURE 25

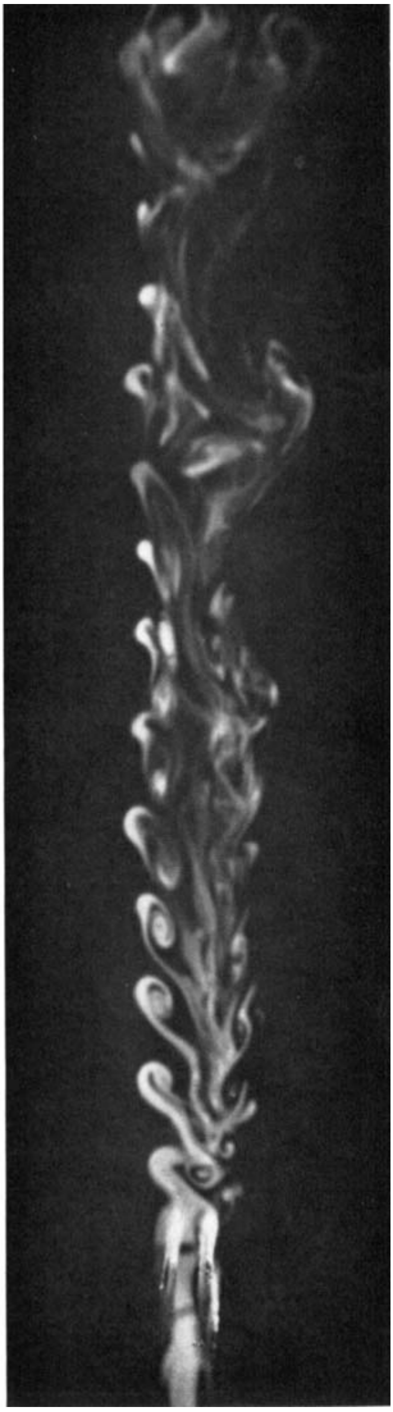

FIGURE 27

FIGURE 24. Symmetric oscillations preceding the transition to turbulence; near-maximum spacing of vortex rows (5; $1\frac{1}{4}$, $1\frac{1}{4}$).

FIGURE 25. The same; near-minimum spacing of vortex rows (5; $1\frac{1}{4}$ $1\frac{1}{4}$).

FIGURE 27. Asymmetric transition to turbulence for the inclined configuration 5; $1\frac{1}{4}$, $1\frac{1}{4}$.

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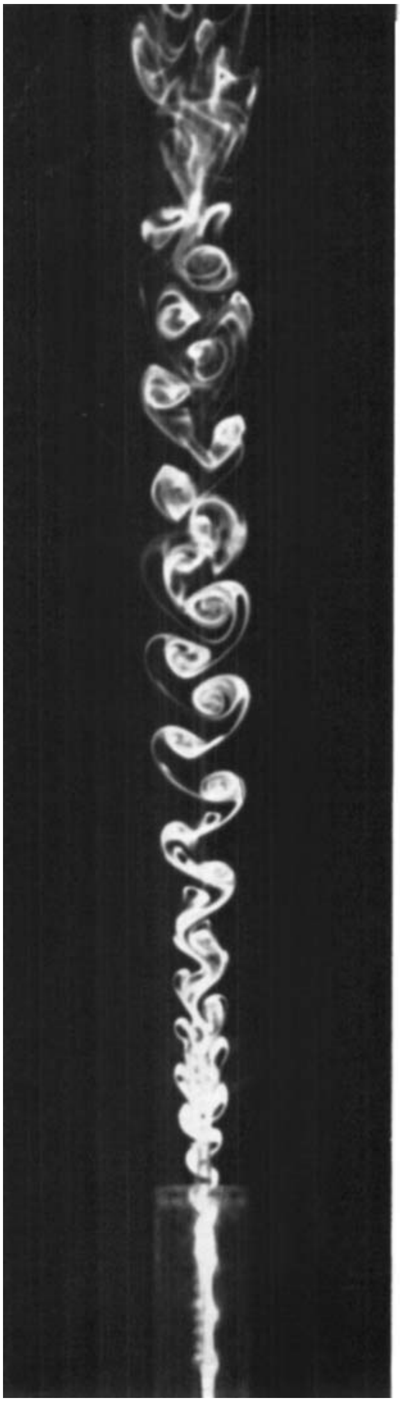

FIGURE 26

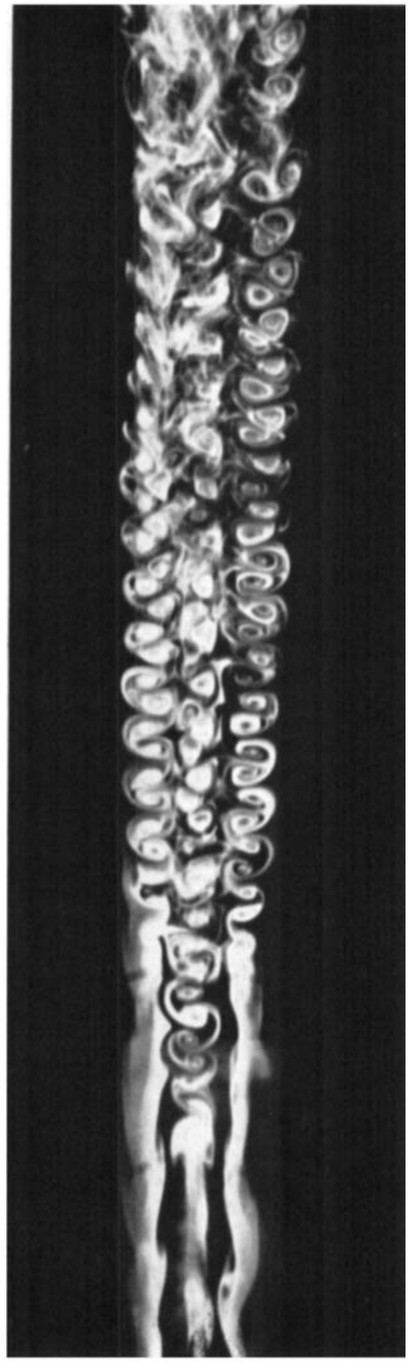

FIGURE 28

FIGURE 26. Transition to turbulence in the new vortex street downstream (11; 1, 1).
FIGURE 28. Transition to turbulence of three vortex streets behind a slightly inclined configuration 19; 5, 5.

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